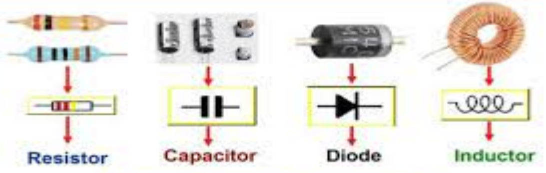
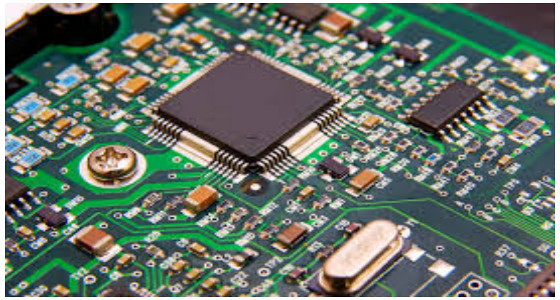
Electronics 1

BSC 113

Fall 2022-2023

Lecture 5





Mesh analysis, Superposition, Thevenin's & Norton theorems

INSTRUCTOR

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➤ Contents

➢ Terms of describing circuits

Name	Definitions
Node	A point where two or more circuit elements join
Essential node	A node where three or more circuit elements join
Branch	A path that connects two nodes
Essential branch	A path which connects two essential nodes without passing through an essential node
Loop	A path whose last node is the same as the starting node
mesh	A loop that does not enclose any other loops

➢ Example 1

➢ For the circuit in the figure, identify

a) all nodes.

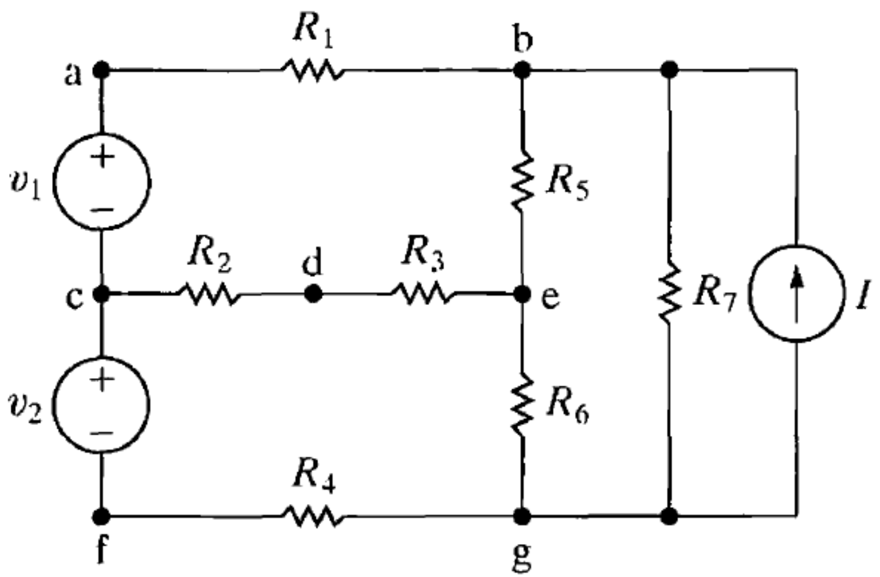
b) all essential nodes.

c) all branches.

d) all essential branches.

e) all meshes.

f) two loops that are not meshes.



➢ Example 1

a) The nodes are a, b, c, d, e, f, and g.

b) The essential nodes are b, c, e, and g.

c) The branches are v1, v2, R1, R2, R3, R4, R5, R6, R7 and I.

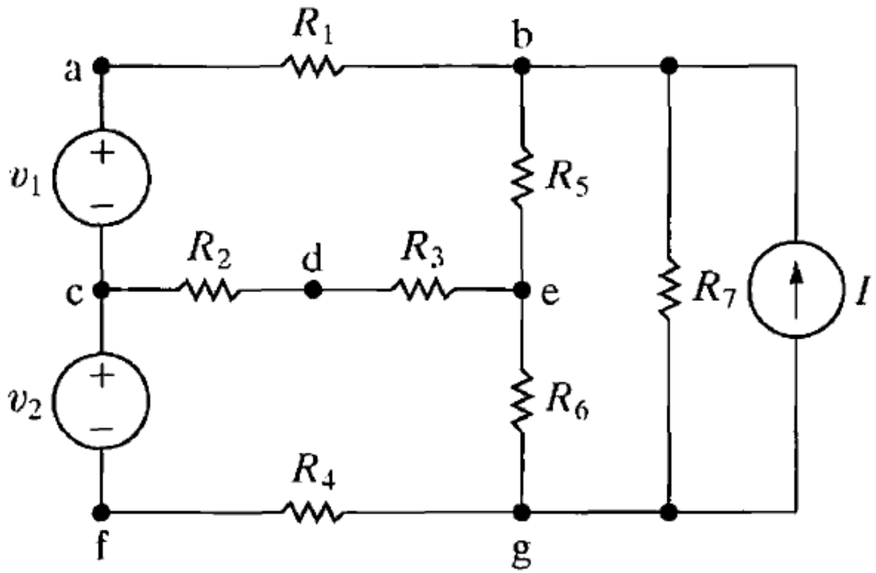
d) The essential branches are:

v1 – R1 ,

R2 – R3 ,

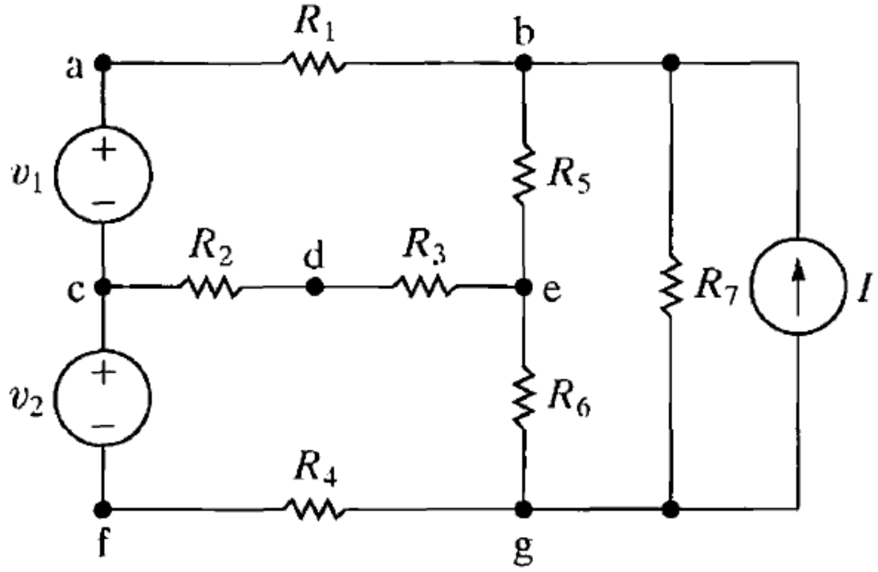
v2 – R4 ,

R5, R6, R7 and I



➢ Example 1

e) The meshes are:
$v_1 - R_1 - R_5 - R_3 - R_2$,
$v_2 - R_2 - R_3 - R_6 - R_4$,
$R_5 - R_7 - R_6$ *and*
$R_7 - I$



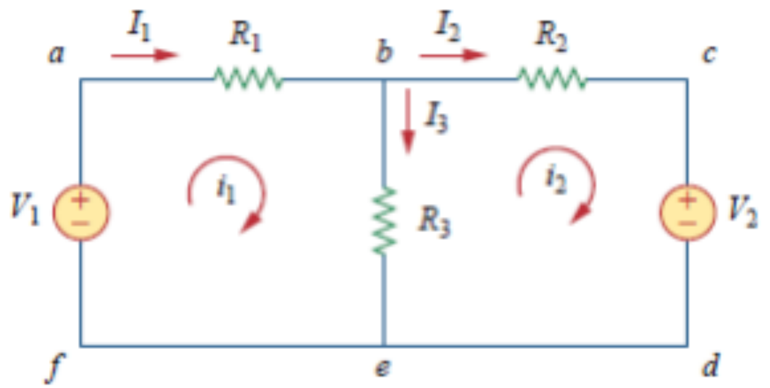
f) The two loops that are not meshes are $v_1 — R_1 — R_5 – R_6$ - R_4 - v_2 and $I — R_5 — R_6$, because there are two loops within them.

Mesh Analysis Method

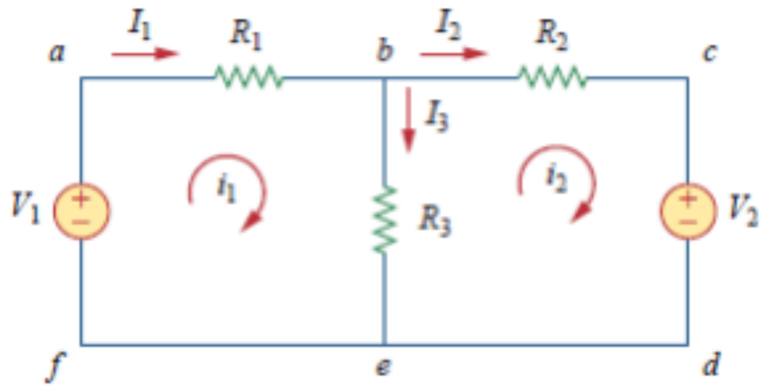
❑ Mesh analysis

- In the mesh analysis of a circuit with n meshes, we take the following three steps.

  1. Assign mesh currents $i_1, i_2, \dots, i_n$ to the n meshes.
  2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
  3. Solve the resulting n simultaneous equations to get the mesh currents.



❑ Mesh analysis



- As shown in figure

loop 1:

$$-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0$$

loop 2:

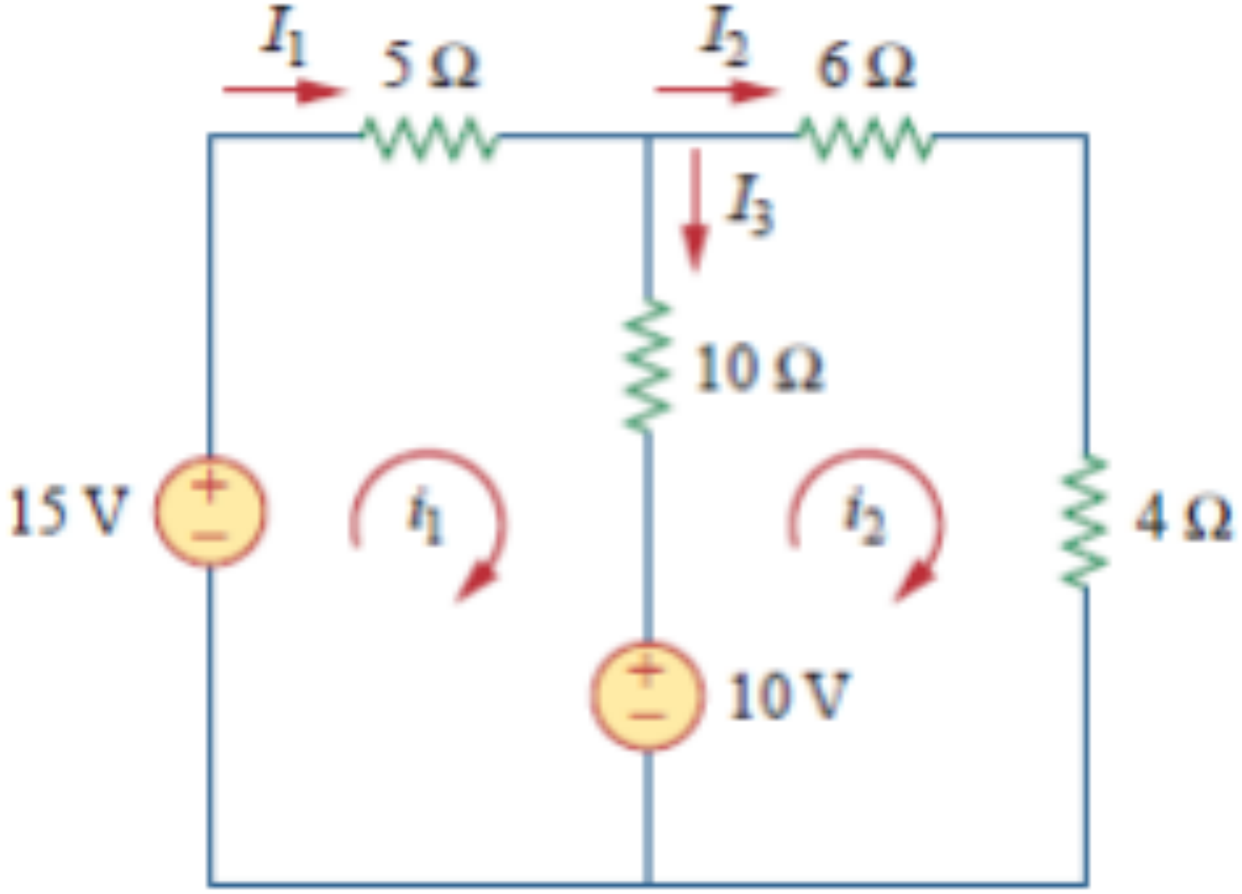
$$R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0$$

After we will solve the two equation we can find:

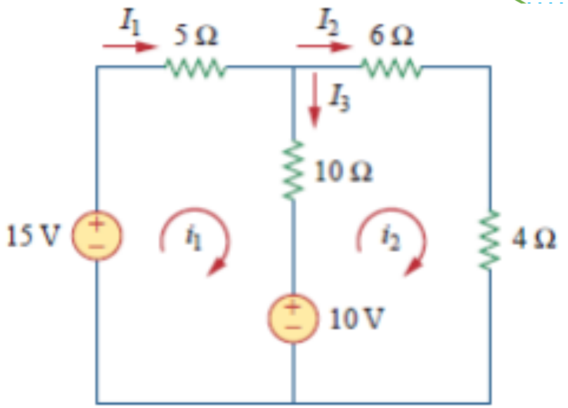
$$I_1 = i_1, \quad I_2 = i_2 \quad and\, I_3 = i_1 - i_2$$

❑ Example 2

- Find the branch currents I_1, I_2 and I_3 using mesh analysis.



❑ Example 2 solution:



- Answer: We first obtain the mesh currents using KVL.

For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0 \qquad (1)$$

and for mesh 2

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0 \qquad (2)$$

from (1) and (2)

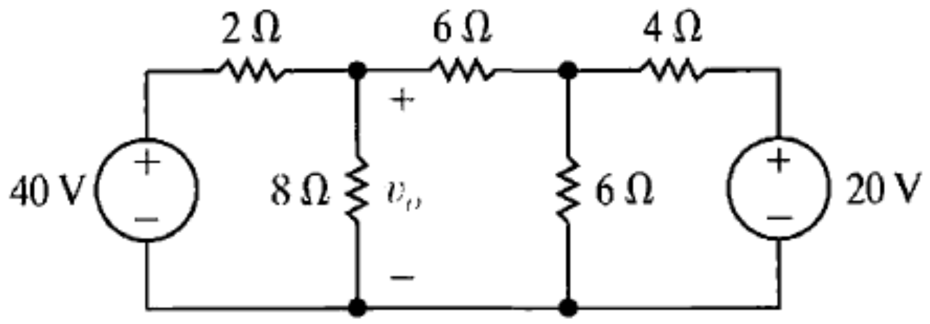
$$i_1 = i_2 = 1A$$

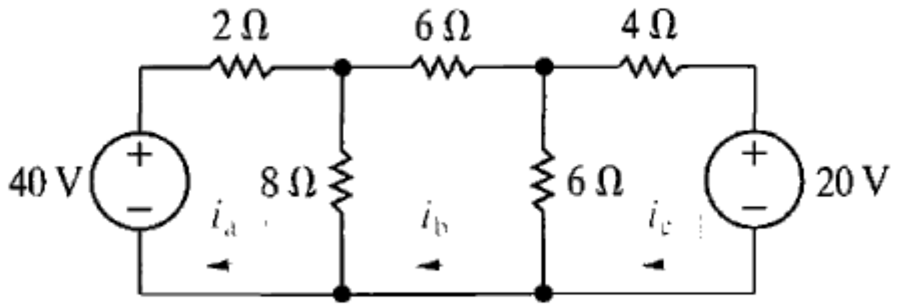
$$I_1 = i_1 = 1A, \quad I_2 = i_2 = 1A \quad and\ I_3 = i_1 - i_2 = 0A$$

❑ Example 3

a) Use the mesh-current method to determine the power associated with each voltage source in the circuit shown.

b) Calculate the voltage v_o across the 8 Ω resistor.





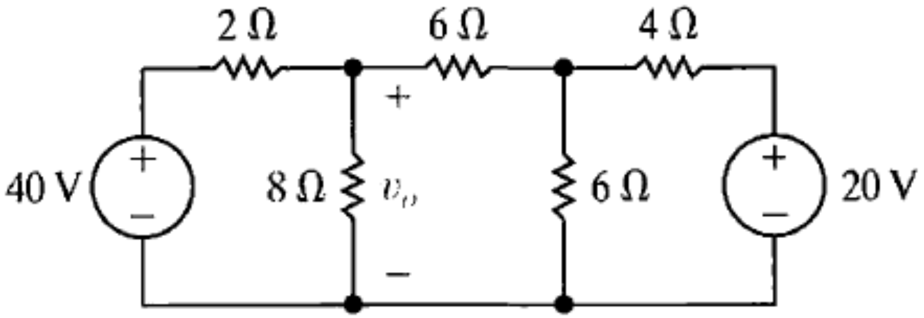
❑ Example 3

a) The three meshes equations are:

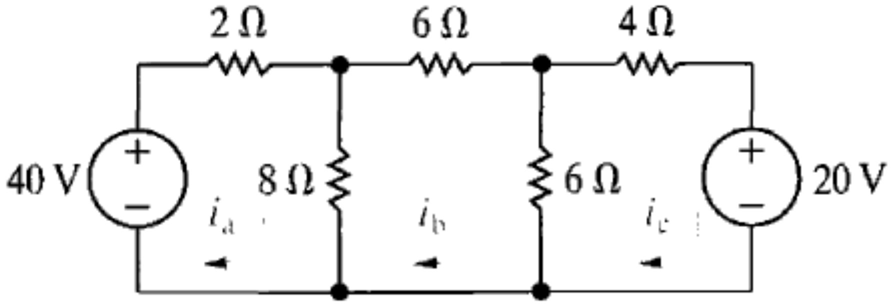
$-40 + 2i_a + 8(i_a - i_b) = 0$

$8(i_b - i_a) + 6\,ib + 6(i_b - i_c) = 0$

$6(i_c - i_b) + 4\,i_c + 20 = 0$



Therefore, the three mesh currents are
$i_a = 5.6$ A, $i_b = 2.0$ A, $i_c = -0.80$ A.

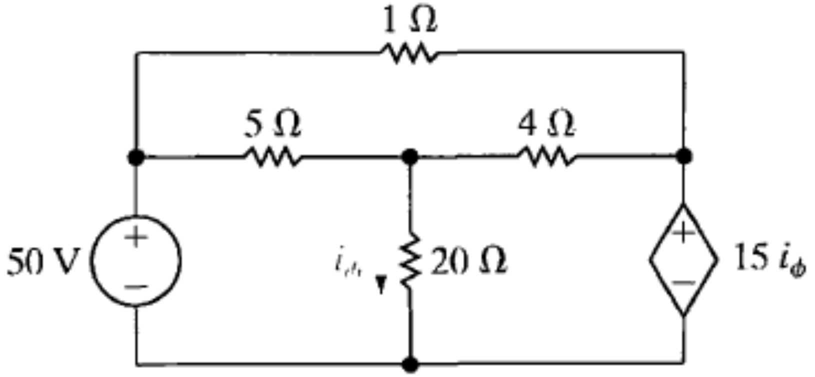


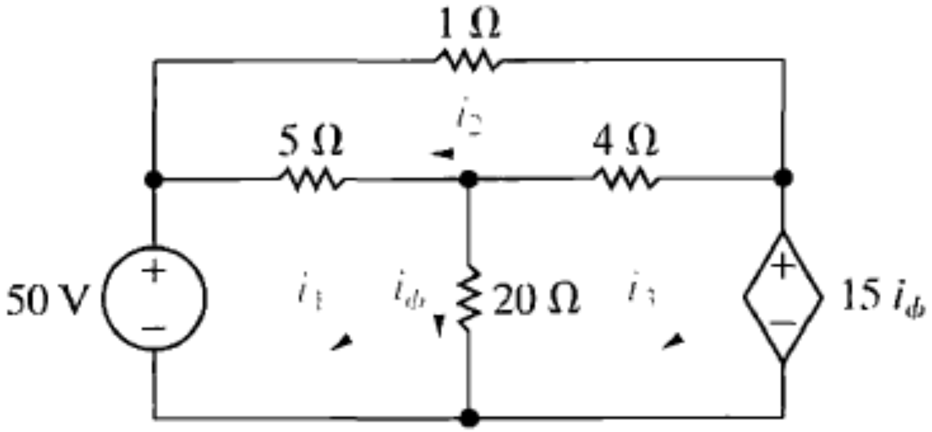
The power associated with each voltage source:
$P_{40V} = -40i_a = -224$ W, $P_{20V} = 20i_c = -16$ W.

b) $v_o = 8(i_a - i_b) = 8(3.6) = 28.8$ V.

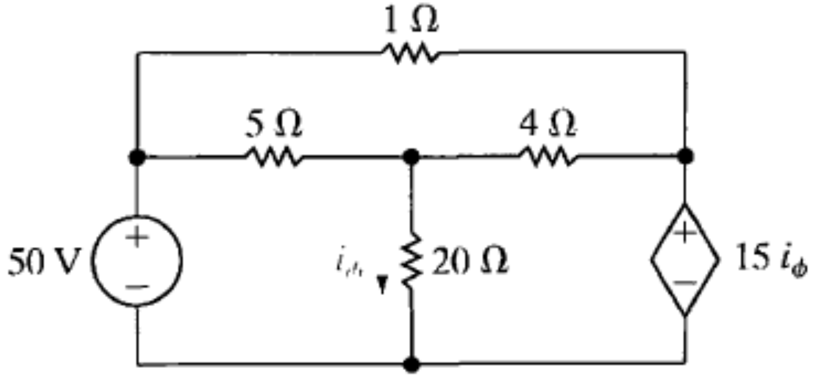
❑ Example 4

Use the mesh-current method of circuit analysis to determine the power dissipated in the 4Ω resistor in the circuit shown.





❑ Example 4



The three mesh-current equations are:

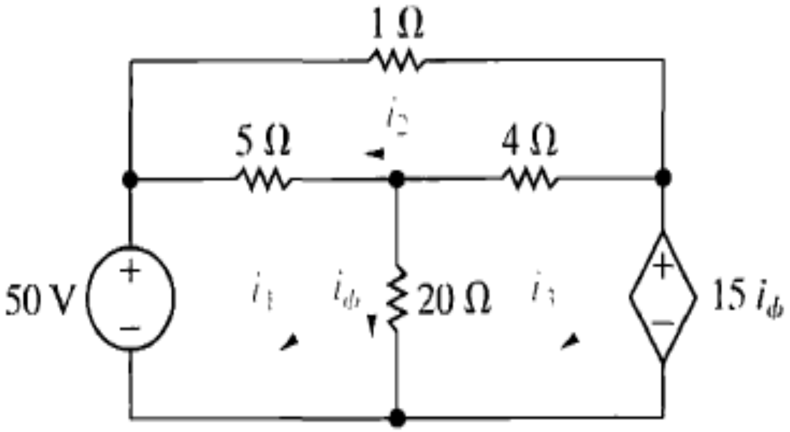
$$5(i_1 - i_2) + 20(i_1 - i_3) = 50$$
$$5(i_2 - i_1) + i_2 + 4(i_2 - i_3) = 0$$
$$20(i_3 - i_1) + 4(i_3 - i_2) + 15i_\Phi = 0$$

We now express the branch current controlling the dependent voltage source in terms of the mesh currents as:

$$i_\Phi = i_1 - i_3$$

Therefore, the mesh currents are:

$i_1 = 29.6$ $\quad$ $i_2 = 26$ A $\quad$ $i_3 = 28$ A

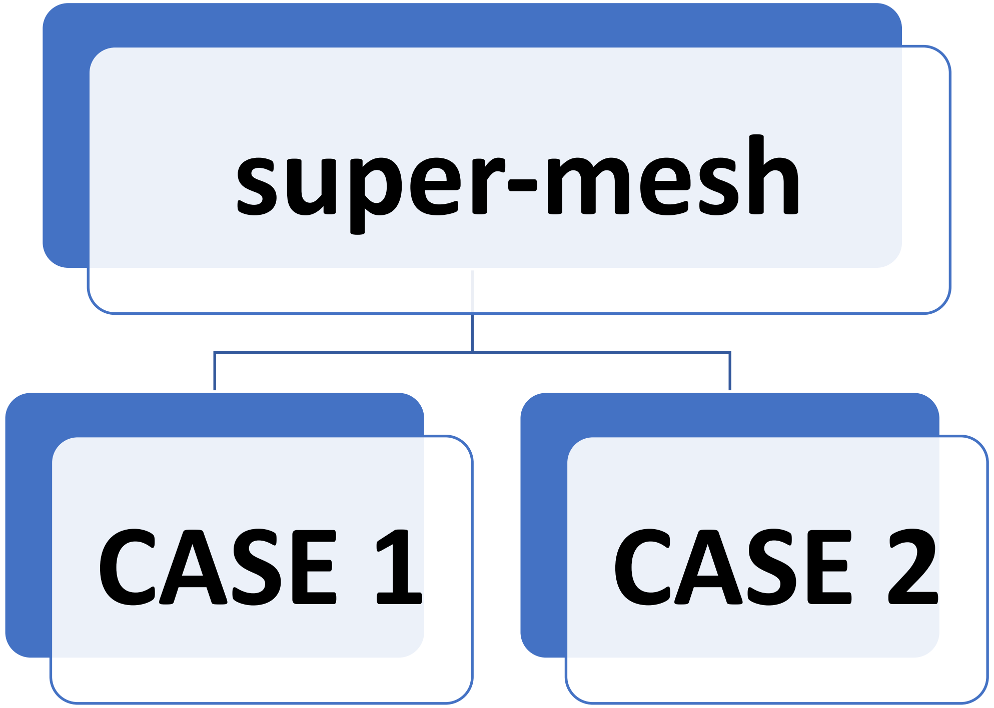


and

$P_{4\Omega} = (i_3 - i_2)^2(4) = (2)^2(4) = 16$ W.

❑ super-mesh

- Mesh Analysis with Current Sources is called super-mesh (A super-mesh results when two meshes have a (dependent or independent) current source in common) and considers as special case.

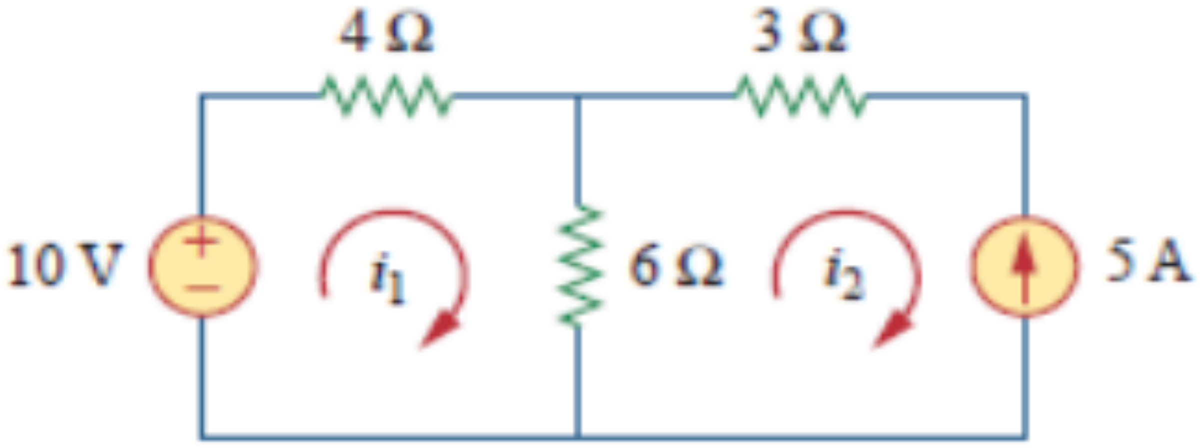


❑ CASE 1

- When a current source exists only in one mesh
- Consider the circuit in next figure, for example. We set $i_2 = -5$ A and write a mesh equation for the other mesh in the usual way; that is,

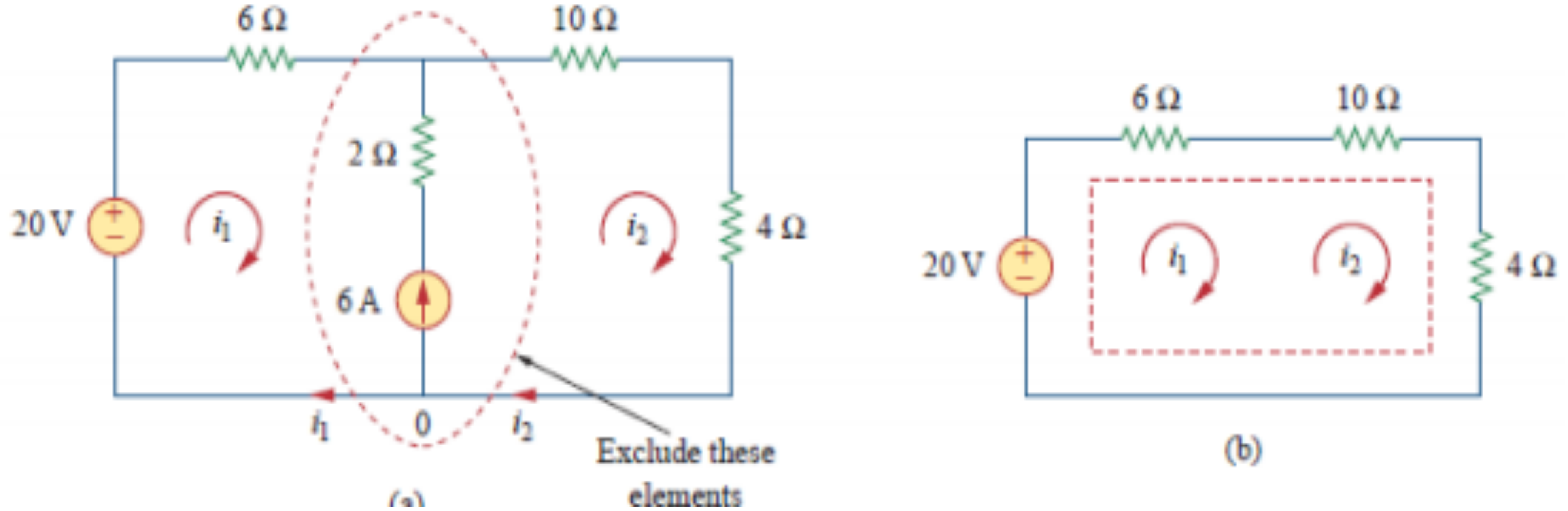
$$-10 + 4i_1 + 6\,(\,i_1 - (-5)\,) = 0 \quad \rightarrow i_1 = -2A$$

- Thus, our analysis is somewhat simplified by this knowledge of the voltage at this node.

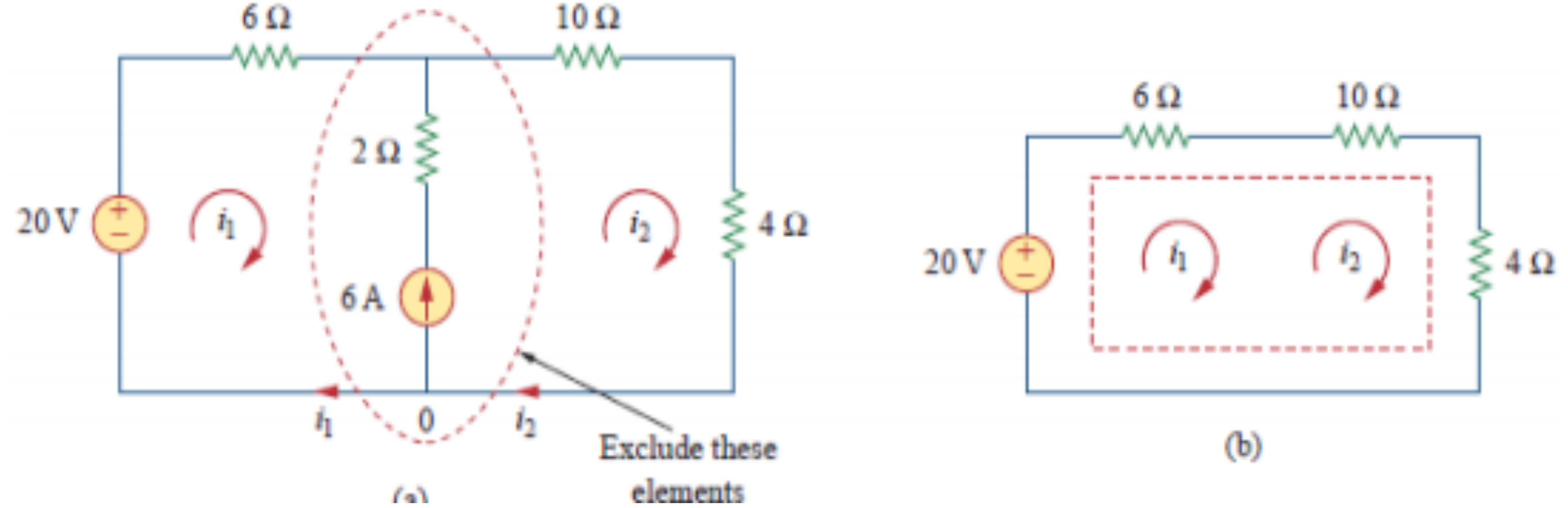


❑ CASE 2

- When a current source exists between two meshes
- Consider the circuit in next figure, for example. We create a super-mesh by excluding the current source and any elements connected in series with it.



❑ CASE 2



$$i_2 - i_1 = 6 \qquad (1)$$

$$-20 + 6\,i_1 + 10\,i_2 + 4\,i_2 = 0 \qquad (2)$$

from (1) and (2)

$$i_1 = -3.2\text{ A}, \qquad i_2 = 2.8\text{ A}$$

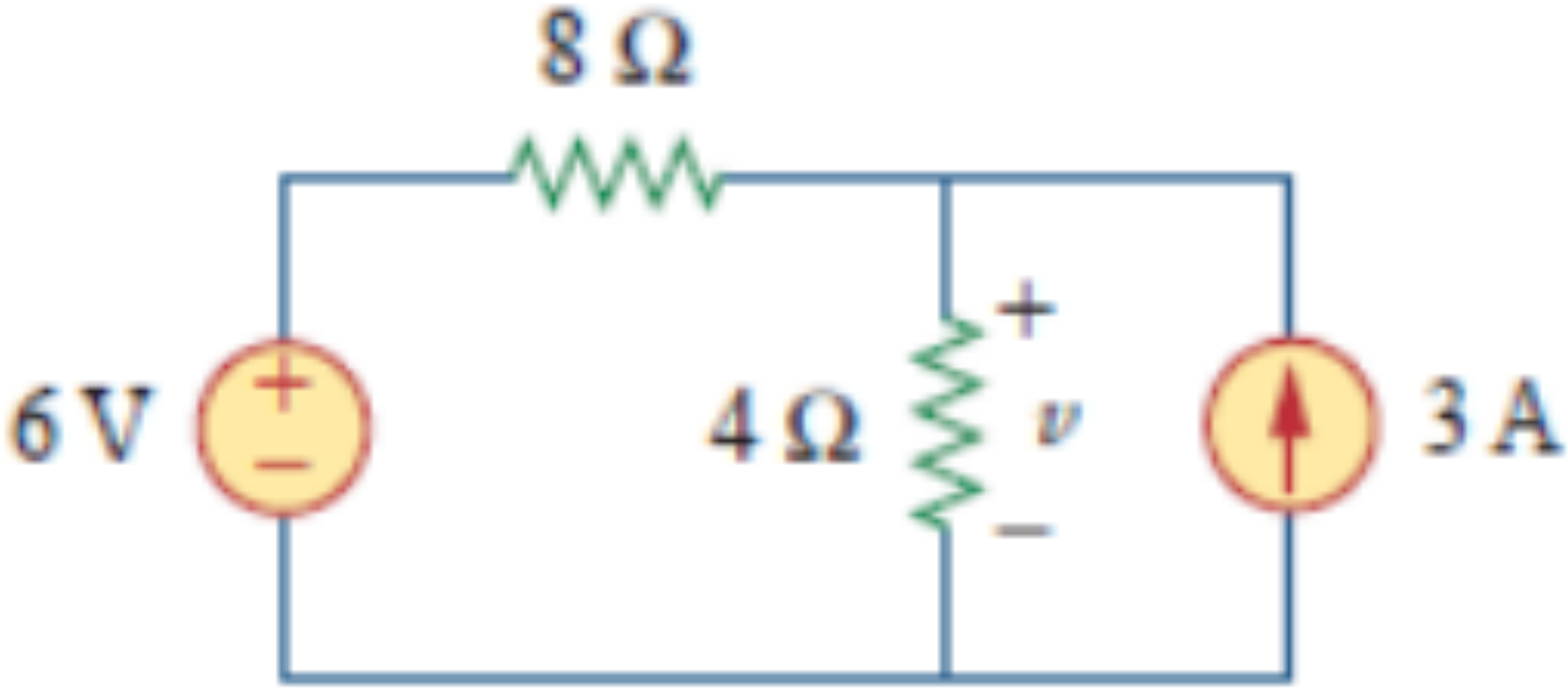
Superposition Method

❑ Superposition

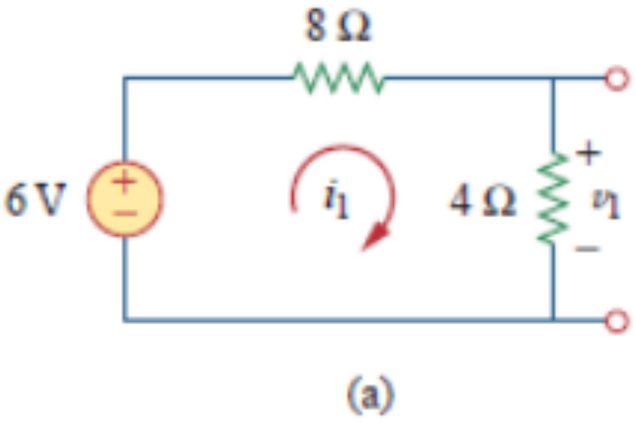
- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone. With these in mind, we apply the superposition principle in three steps:
  1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source.
  2. Repeat step 1 for each of the other independent sources.
  3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

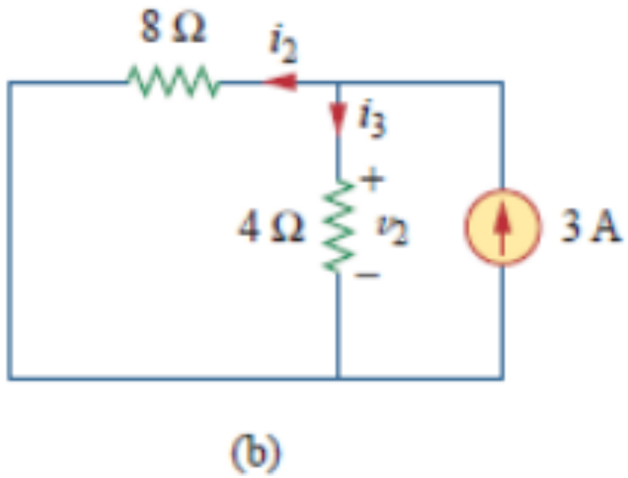
❑ Example5:

- Use the superposition theorem to find v in the circuit



❑ Example5:





Answer: $v = v_1 + v_2$

from figure a by voltage divider

$$v_1 = 4i_1 = \frac{4}{4+8} * 6 = 2V$$

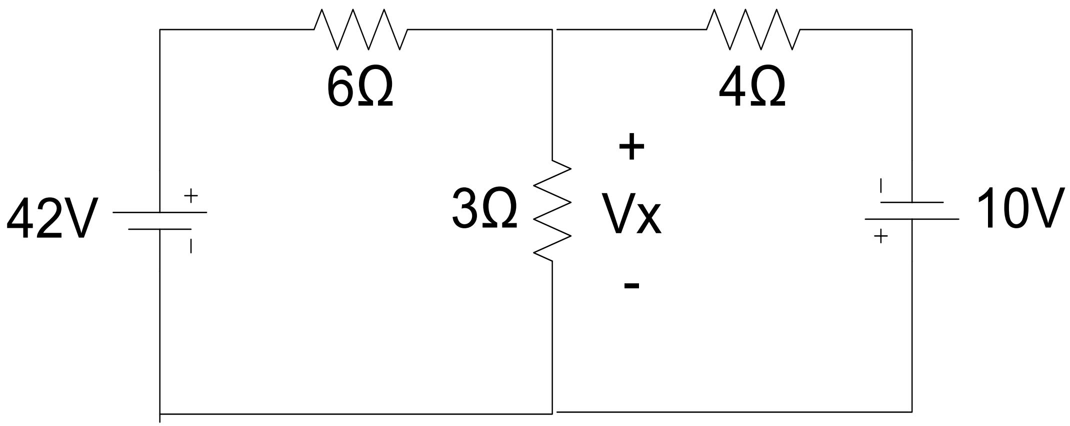
from figure b by current divider

$$v_2 = 4i_3 = 4 * \frac{8}{4+8} * 3 = 8V$$

$$v = v_1 + v_2 = 2 + 8 = 10V$$

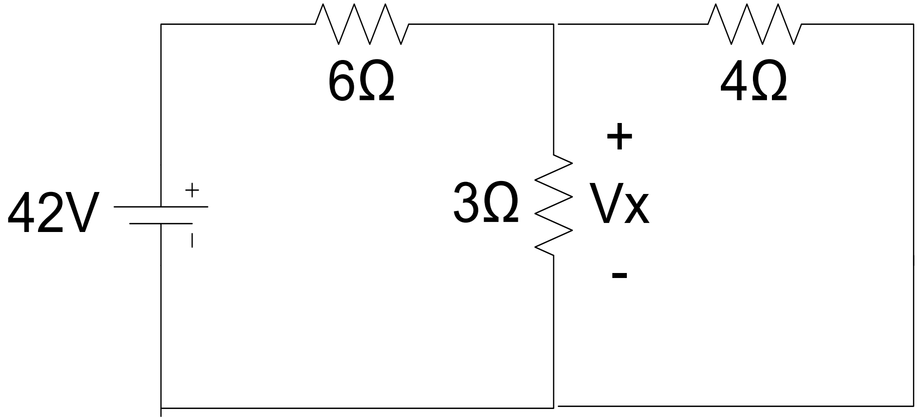
❑ Example 6

- Find voltage Vx using superposition theorem



❑ Example 6

- Find voltage Vx using superposition theorem



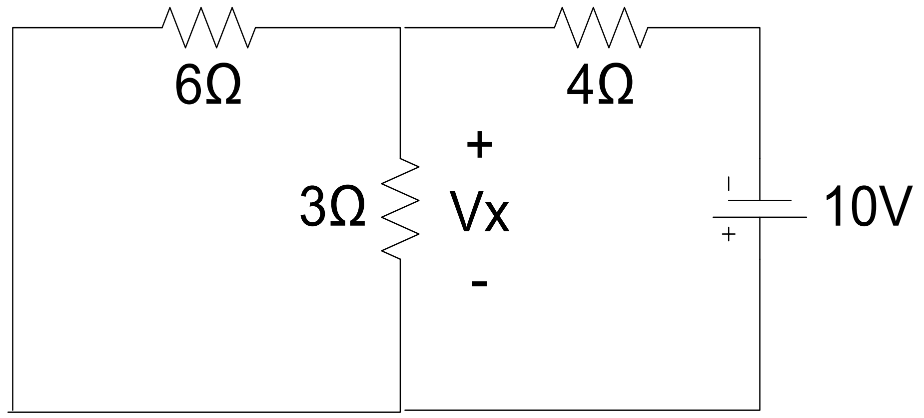
$$Considering\,42V source only (10 V source SC)$$

$$Vx_{(42V)} = \frac{(3 \| 4)}{6+(3 \| 4)} \times 42 = \frac{(12/7)}{6+(12/7)} \times 42$$

$$= 9.333V$$

❑ Example 6

- Find voltage Vx using superposition theorem



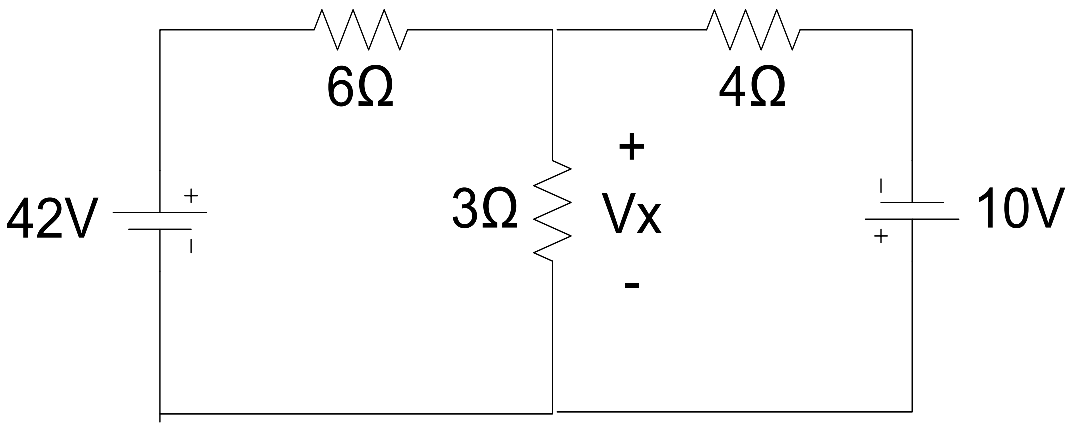
$$Only\,10V\,source\,connected\,(42V\,source\,replaced\,by\,SC)$$

$$Vx_{(10V)} = -\frac{(6\parallel 3)}{(6\parallel 3)+4}\times 10 = -\frac{2}{2+4}\times 10$$

$$= -3.333V$$

❑ Example 6

➢ Find voltage Vx using superposition theorem



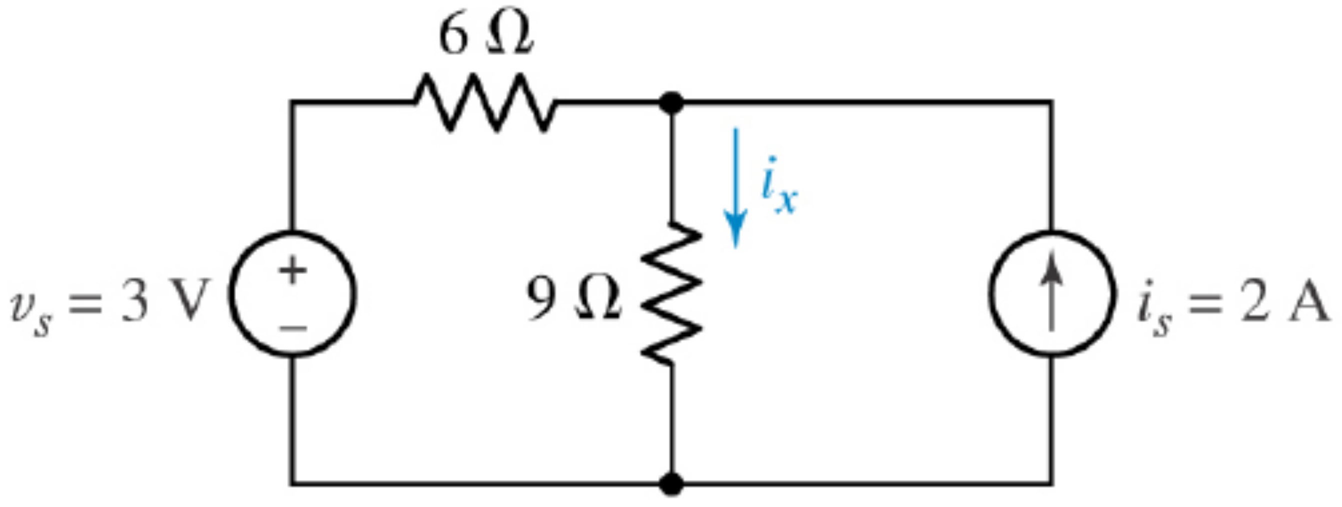
$$TotalVoltage =$$

$$Vx = Vx_{(42V)} + Vx_{(10V)}$$

$$= 9.333 - 3.333 = 6V$$

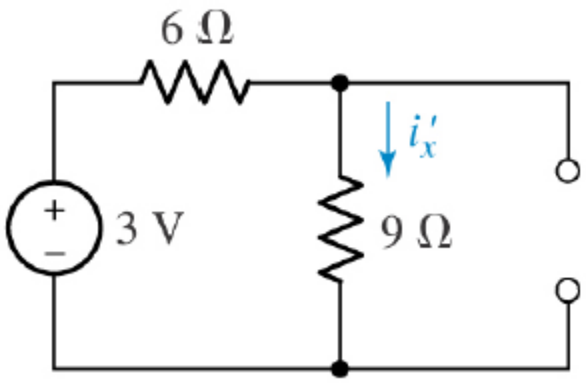
❑ Example 7

- Use superposition to find ix

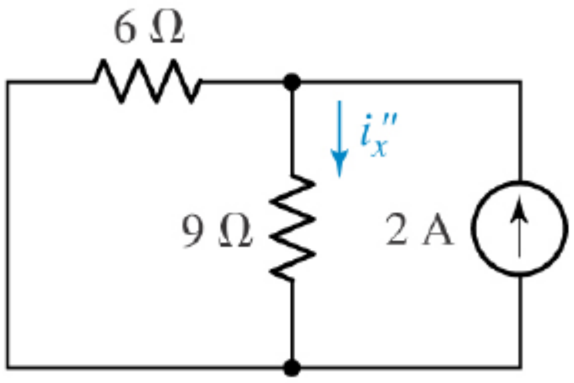


(a)

❑ Example 7



(b)



(c)

Step 1:

Only 3V source connected(2A source is OC)

$$i_x^{'} = 3/15 = 0.2\ \mathrm{A}$$

Step2:

Only 2A source connected(3V source is SC)

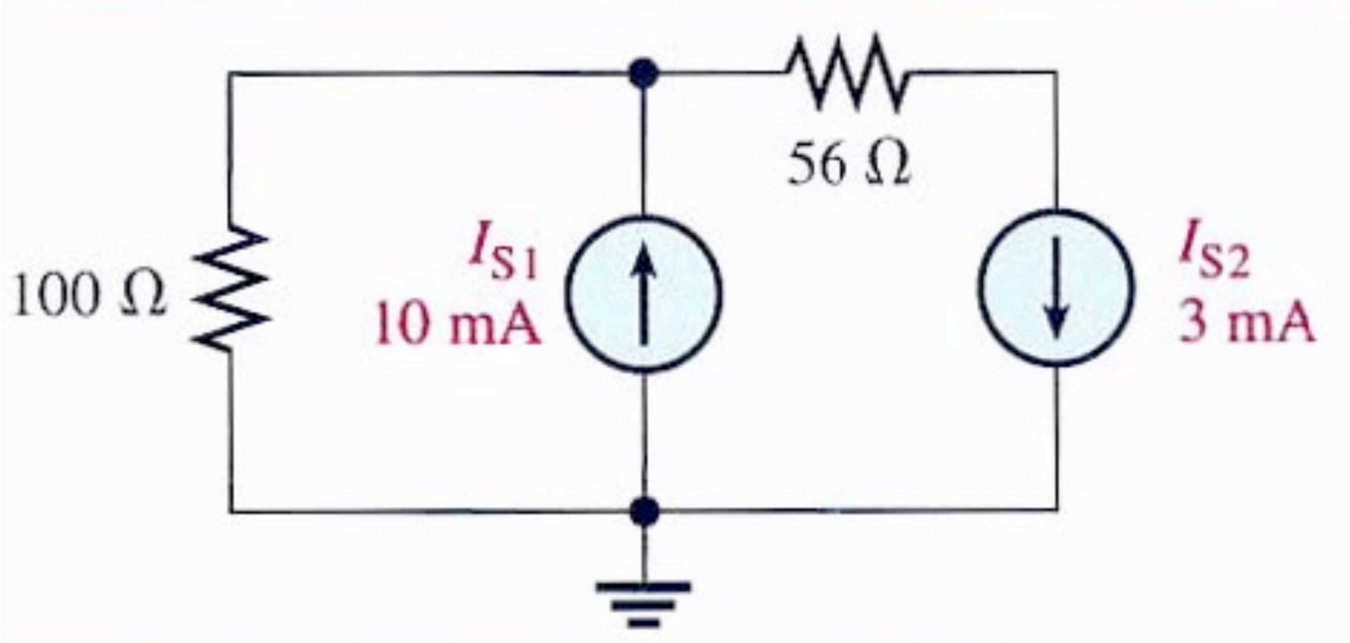
$$i_x^{''} = 2x6/(6+9) = 0.8\ A$$

Step 3:Totalcurrent=0.2+0.8=1A

$$i_x = 1.0\ A$$

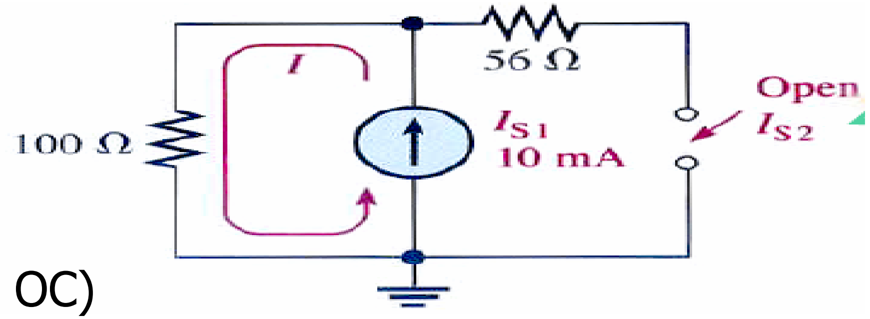
❑ Example 8

➢ Find the current through 100-ohm resistor



❑ Example 8

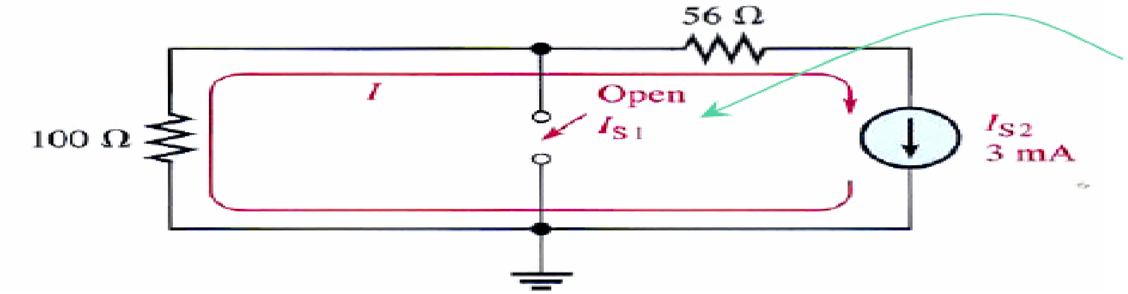
➢ Find the current through 100-ohm resistor



Step 1:

Only 10ma source connected in circuit(3mA source OC)

I=10mA



Step2: Only 3mA source connected(10mA source OC)

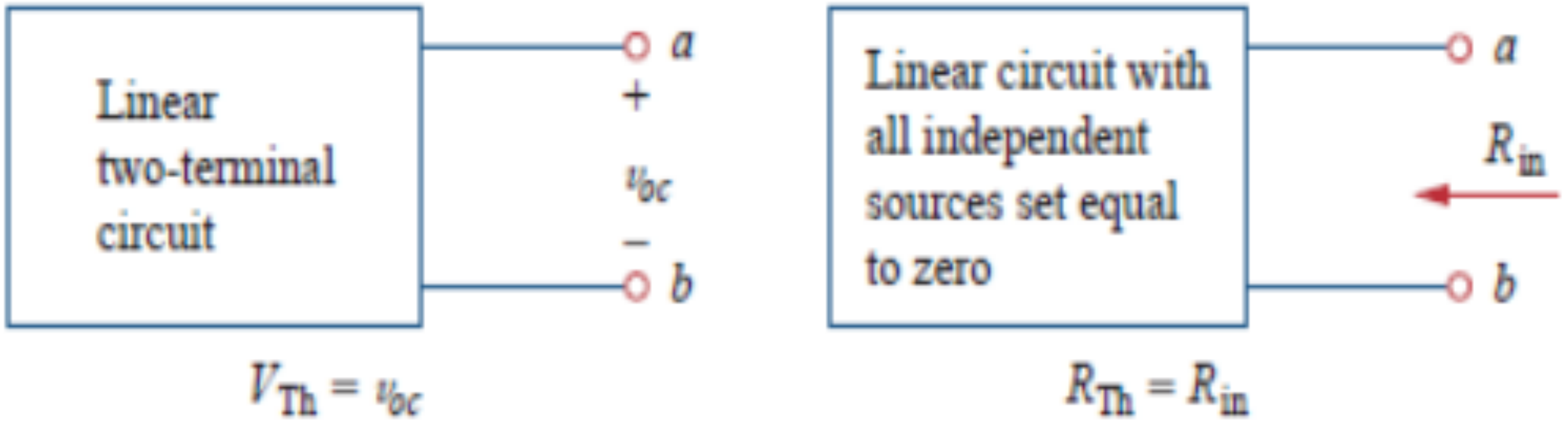
I’=3mA

Total Current:10mA-3mA=7mA

Thevenin’s theorem

❑ Thevenin's theorem

➢ Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off as shown in figure



THEVENIN & NORTON

THEVENIN'S THEOREM:

Consider the following:

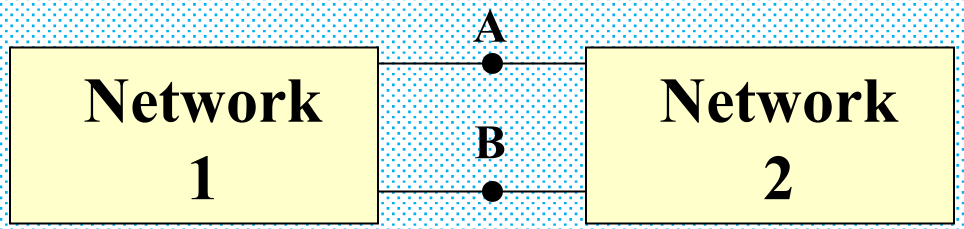


Figure: Coupled networks.

For purposes of discussion, at this point, we consider that both networks are composed of resistors and independent voltage and current sources

THEVENIN & NORTON

THEVENIN'S THEOREM:

Suppose Network 2 is detached from Network 1 and we focus temporarily only on Network 1.

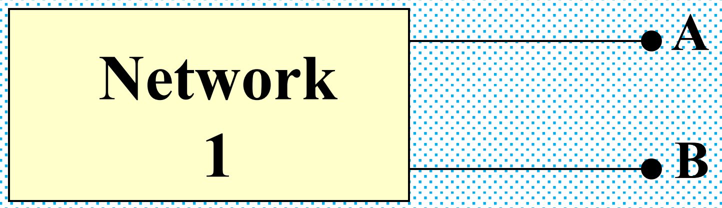
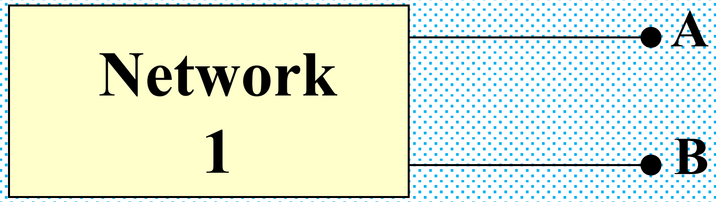


Figure: Network 1, open-circuited.

Network 1 can be as complicated in structure as one can imagine. Maybe 45 meshes, 387 resistors, 91 voltage sources and 39 current sources.

THEVENIN & NORTON

THEVENIN'S THEOREM:



Now place a voltmeter across terminals A-B and read the voltage. We call this the open-circuit voltage.

No matter how complicated Network 1 is, we read one voltage. It is either positive at A, (with respect to B) or negative at A.

We call this voltage V_{os} and we also call it $V_{THEVENIN} = V_{TH}$

THEVENIN & NORTON

THEVENIN'S THEOREM:

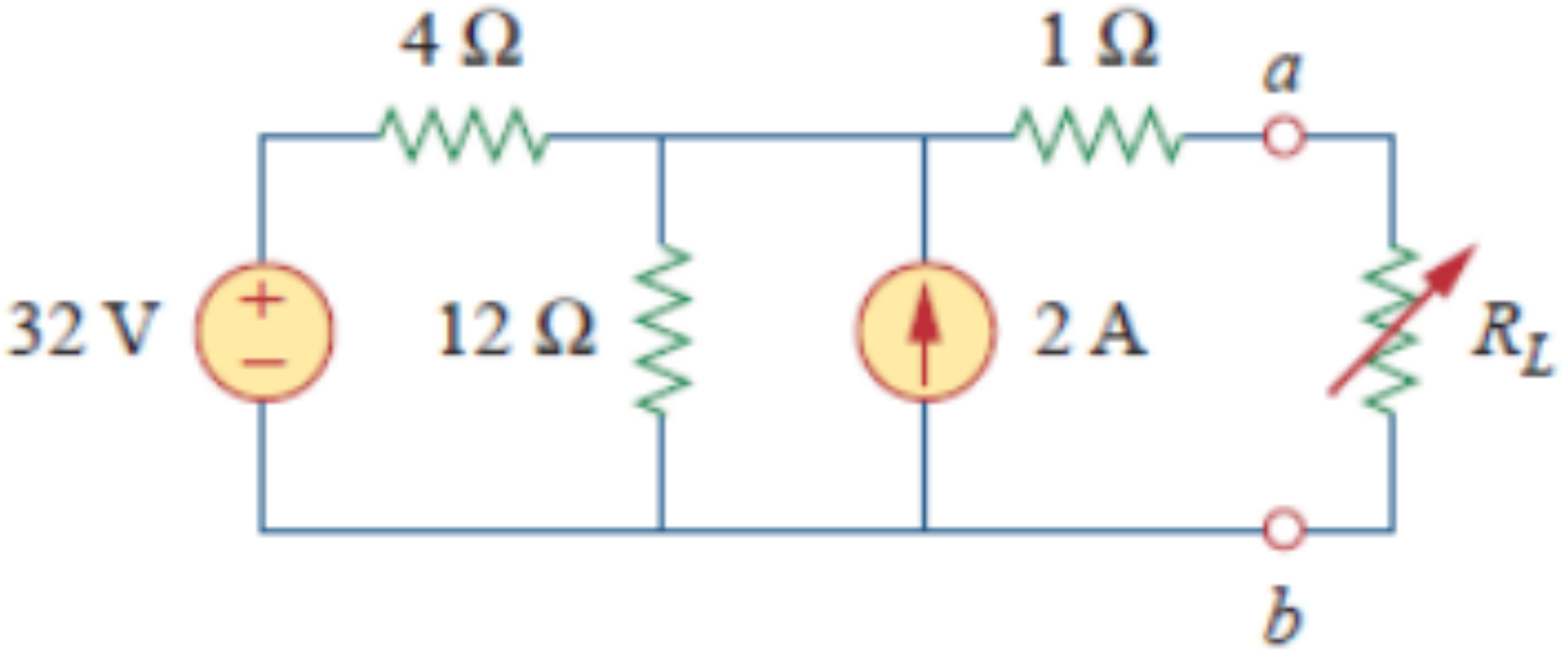
- **We now deactivate all sources of Network 1.**

- **To deactivate a voltage source, we remove the source and replace it with a short circuit.**

- **To deactivate a current source, we remove the source.**

❑ Example1

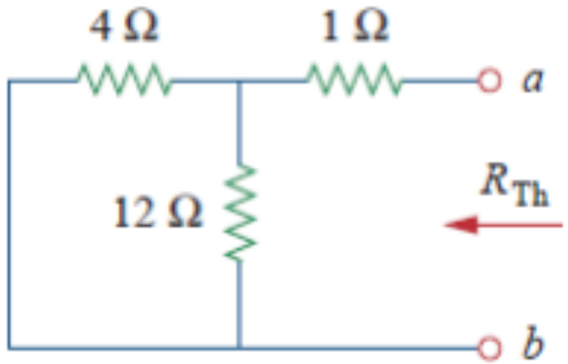
➢ Find the Thevenin equivalent circuit of the circuit shown, to the left of the terminals a - b.



❑ Example1

- Find the Thevenin equivalent circuit of the circuit shown, to the left of the terminals a - b.

Answer:



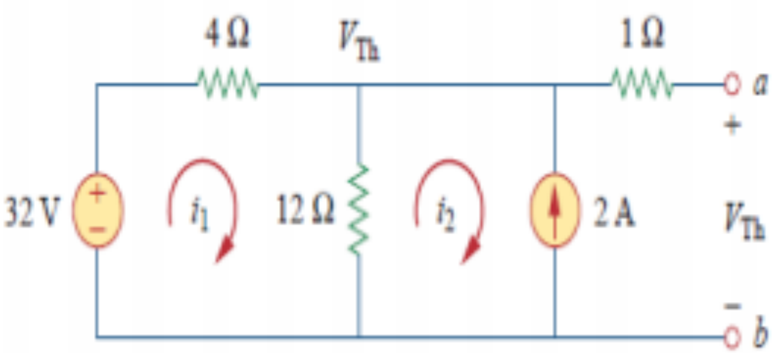
R_{th}= (4//12) +1= 4Ω

$$i_2 = -2A$$

$$-32 + 16i_1 - 12i_2 = 0$$

$$i_1 = 0.5A$$

$$V_{Th} = 12(i_1 - i_2) = 30V$$



THEVENIN & NORTON

THEVENIN'S THEOREM: Example 2.

Find V_X by first finding V_{TH} and R_{TH} to the left of A-B.

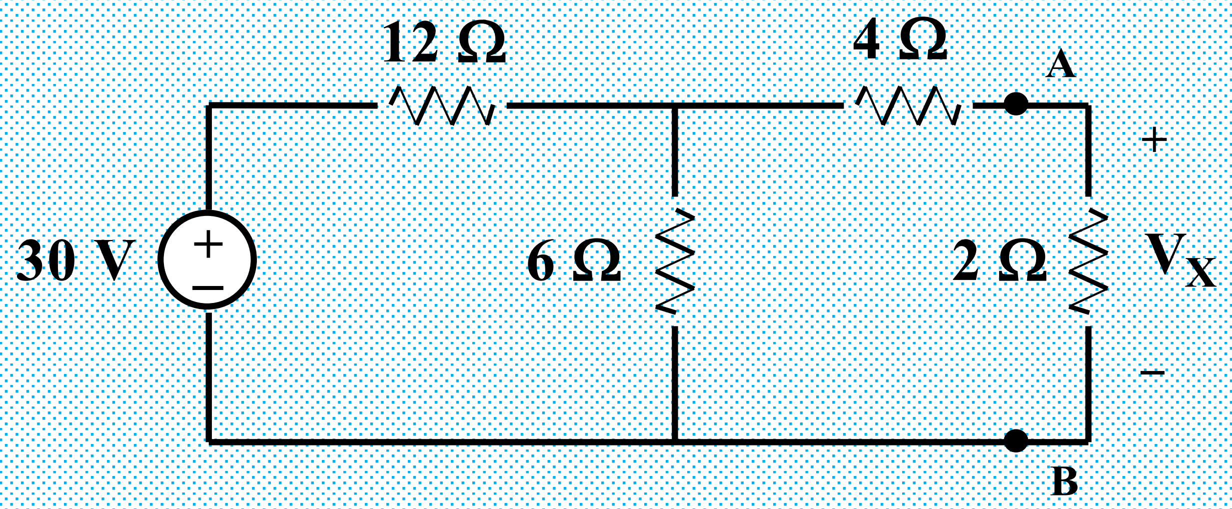


Figure: Circuit for Example 2.

First remove everything to the right of A-B.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 2. continued

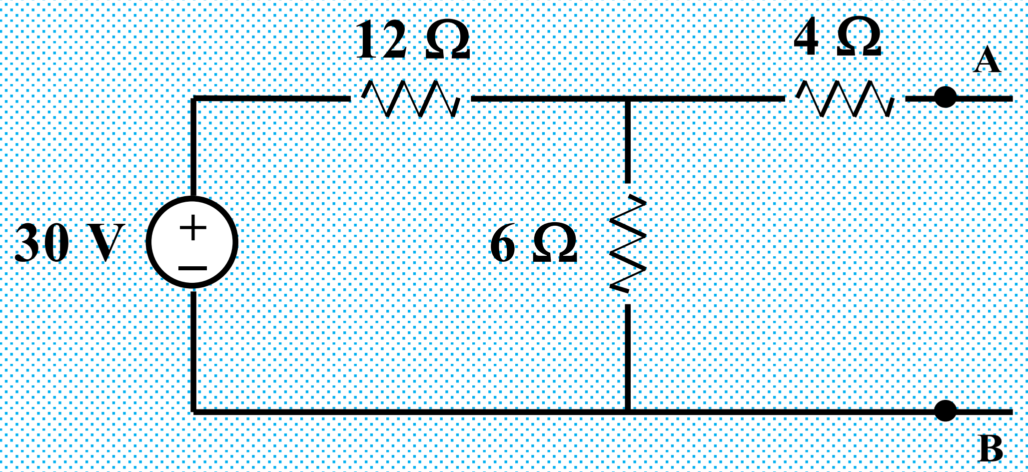


Figure: Circuit for finding V_{TH} for Example 2.

$$V_{AB} = \frac{(30)(6)}{6+12} = 10V$$

Notice that there is no current flowing in the 4 Ω resistor (A-B) is open. Thus, there can be no voltage across the resistor.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 2. continued

We now deactivate the sources to the left of A-B and find the resistance seen looking in these terminals.

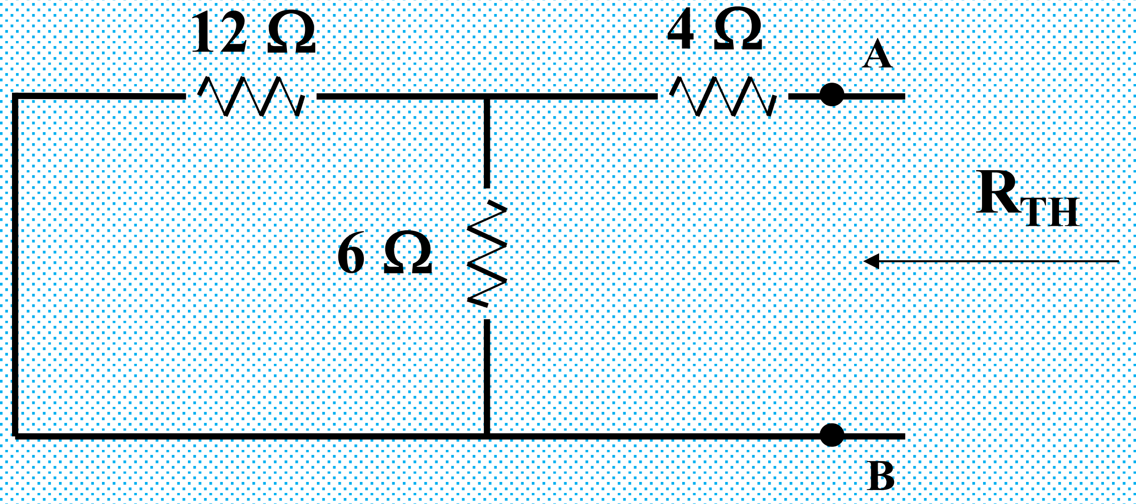


Figure: Circuit for find R_{TH} for Example 2.

We see,

$$R_{TH} = 12||6 + 4 = 8\,\Omega$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 2. continued

After having found the Thevenin circuit, we connect this to the load in order to find V_X.

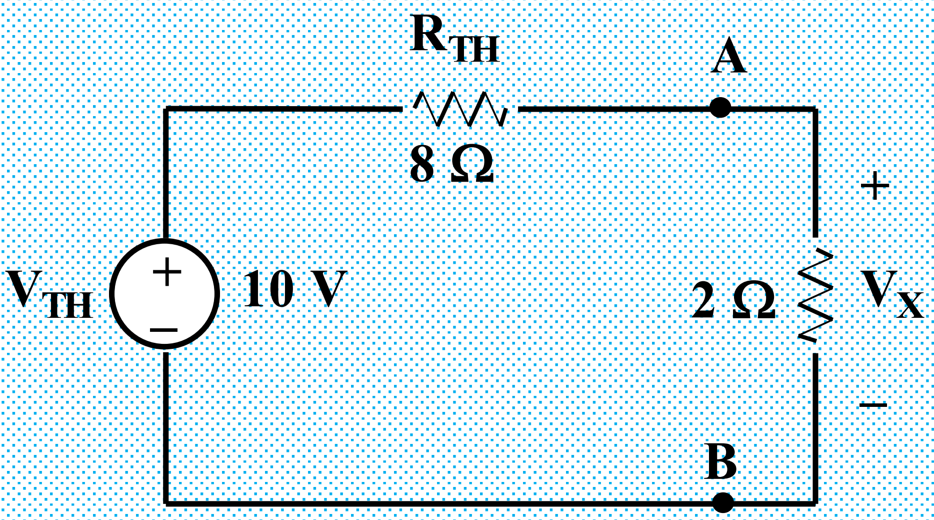


Figure: Circuit of Ex 2 after connecting Thevenin circuit.

$$V_X = \frac{(10)(2)}{2+8} = 2V$$

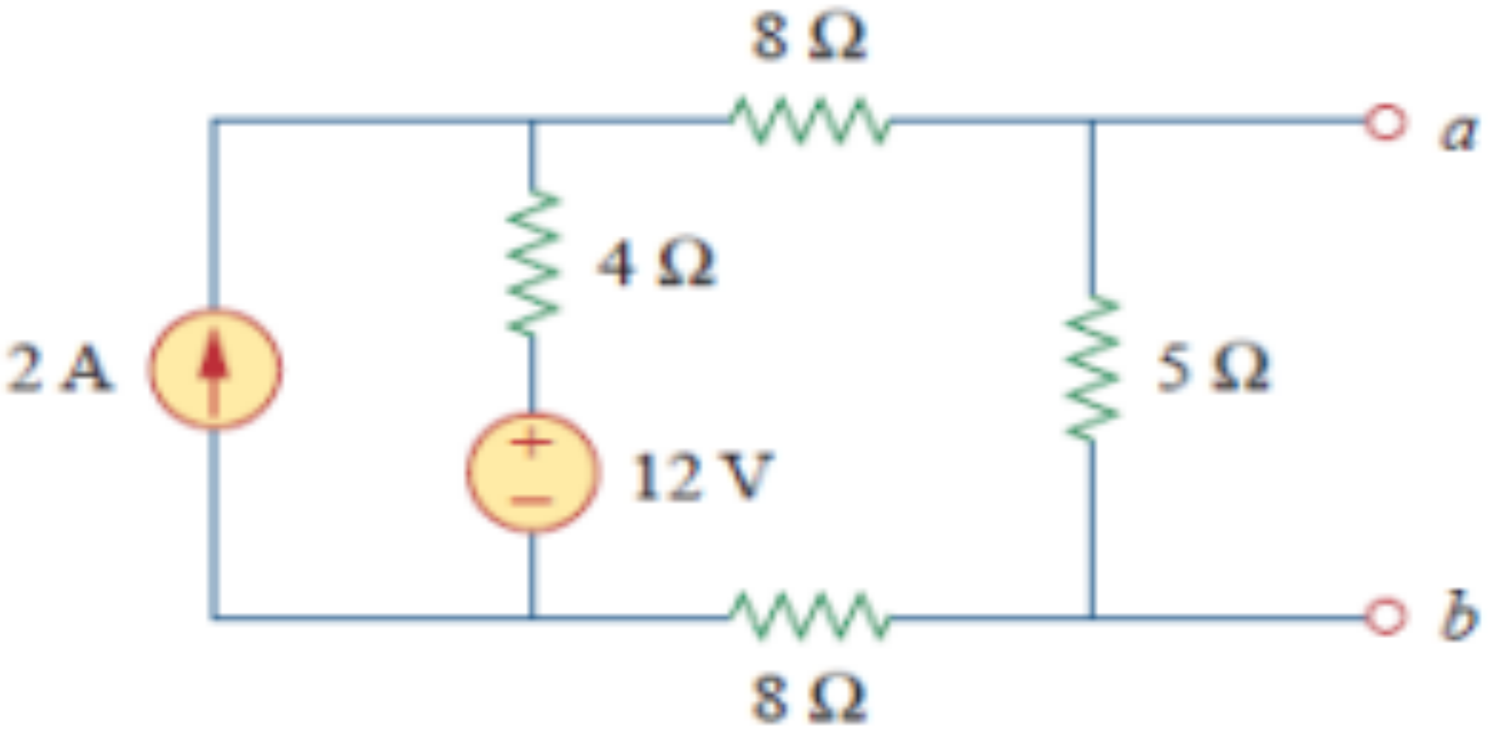
Norton theorem

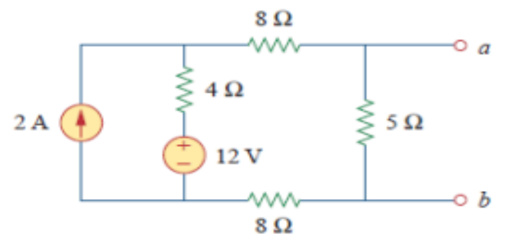
❑ Norton theorem

- Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source $I_N = V_{Th}/R_{Th}$ in parallel with a resistor $R_N = R_{Th}$, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

❑ Example

- Find the Norton equivalent circuit of the circuit shown, to the left of the terminals a - b.





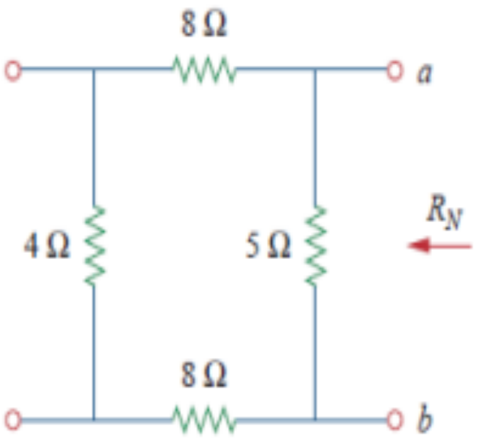
❑ Example

➢ Find the Norton equivalent circuit of the circuit shown, to the left of the terminals a - b.

Answer:

$R_N = 5//(8+4+8) =$

4Ω



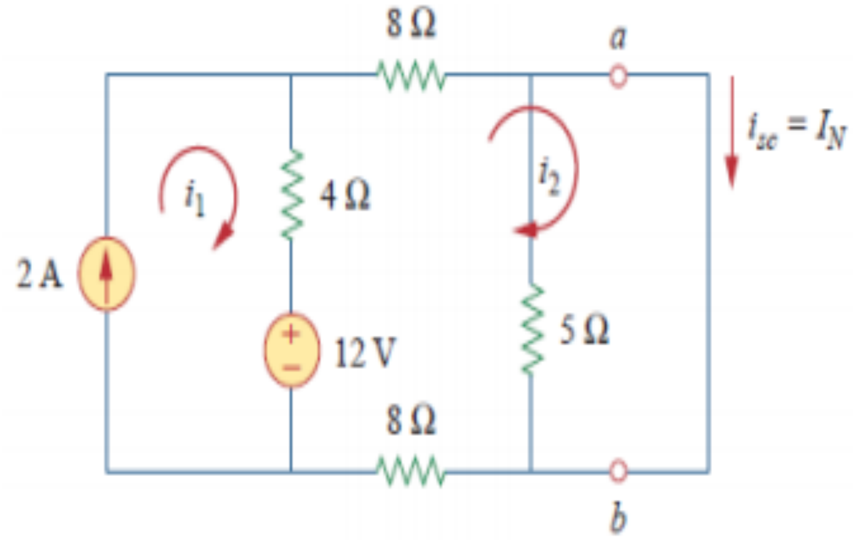
$i_1 = 2A$

$20i_2 - 4i_1 - 12$

$= 0$

$i_2 = 1A = i_{sc}$

$= I_N$




Thank
you